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# CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS INVOLVING GENERALIZED DIFFERENTIAL AND INTEGRAL OPERATORS

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Abstract. The aim of the present paper is to obtain some interesting subordination results for certain subclasses of analytic functions associated with generalized Differential and Integral operator.

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## 1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic and univalent in the open unit disc $\mathcal{U} = \{z : z \in \mathcal{C} \text{ and } |z| < 1\}$. Let $\mathcal{K}$ denote the familiar class of functions $f \in \mathcal{A}$ which are univalent and convex in $\mathcal{U}$.

**Definition 1.1.** *(Convolution) Given two functions $f$ and $g$ in the class $\mathcal{A}$, where $f$ is given by ( 1.1) and $g$ is given by $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ the Hadamard product (or convolution) $f * g$ is defined by the power series*

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z) \quad (z \in \mathcal{U}).$$

Now we define the generalized differential operator [10] $\mathcal{D}^k_{\alpha,\lambda,l}$ as follows

$$\mathcal{D}^k_{\alpha,\lambda,l} f(z) = \underbrace{(\Psi(z) * \Psi(z) * ... * \Psi(z))}_{k\,times} * f(z)$$

where

$$\Psi(z) = \frac{1}{l+1}\left[\frac{z(\lambda - \alpha + 1)}{(1-z)^2} - \frac{z(\lambda - \alpha + l)}{(1-z)}\right]$$

and

$$\mathcal{D}^k_{\alpha,\lambda,l} f(z) = z + \sum_{n=2}^{\infty} \left[ \frac{(\lambda-\alpha)(n-1)+l+n}{l+1} \right]^k a_n z^n \tag{1.2}$$

where $\alpha \geq 0,\ \lambda \geq 0, n \in N_0, l \geq 0$.

**Remark 1.2.** *By giving specific values to* $\alpha$, $\lambda$, $l$ *and* $k$ *we obtain the various operators studied earlier by Al - Oboudi* [1], *Catas* [3] , *Cho and Kim* [4], *Cho and Srivastava* [5], *Maslina Darus and Rabha Ibrahim* [6], *Sălăgean* [9], *Uralegaddi and Somanatha* [13].

In [10], using the generalized differential operator we have defined the new subclasses $\mathcal{M}^k_{\alpha,\lambda,l}(\mu)$ and $\mathcal{N}^k_{\alpha,\lambda,l}(\mu)$ as follows

**Definition 1.3.** [10] *A function* $f \in \mathcal{A}$ *is said to be in the class* $\mathcal{M}^k_{\alpha,\lambda,l}(\mu)$ *if it satisfies the inequality*

$$\Re \left\{ \frac{z(\mathcal{D}^k_{\alpha,\lambda,l} f(z))'}{\mathcal{D}^k_{\alpha,\lambda,l} f(z)} \right\} < \mu, \quad (z \in \mathcal{U})$$

*for some* $\mu(\mu > 1)$.

**Definition 1.4.** [10] *A function* $f \in \mathcal{A}$ *is said to be in the class* $\mathcal{N}^k_{\alpha,\lambda,l}(\mu)$ *if it satisfies the inequality*

$$\Re \left\{ \frac{z(\mathcal{D}^k_{\alpha,\lambda,l} f(z))''}{(\mathcal{D}^k_{\alpha,\lambda,l} f(z))'} \right\} < \mu, \quad (z \in \mathcal{U})$$

*for some* $\mu(\mu > 1)$.

We have $f \in \mathcal{N}^k_{\alpha,\lambda,l}(\mu)$ if and only if $zf' \in \mathcal{M}^k_{\alpha,\lambda,l}(\mu)$.

**Remark 1.5.** *The above classes reduce to the subclasses studied by*

(1) *Uralegaddi et. al* [12, 11] , *for* $1 < \mu \leq \frac{4}{3}$ *and* $k = 0$
(2) *Owa and Nishiwaki* [7], *for* $\mu > 1$ *and* $k = 0$
(3) *Bulut* [2], *for* $\mu > 1, \alpha = 1, l = 0$
(4) *M. Darus and R. Ibrahim* [6], *for* $l = 0$.

Analogous to the differential operator $\mathcal{D}^k_{\alpha,\lambda,l}$, we consider the Integral operator $\mathcal{I}^k_{\alpha,\lambda,l}$ [10] defined as follows: Let

$$F(z) = \underbrace{(\Psi(z) * \Psi(z) * ... * \Psi(z))}_{k\,times} = z + \sum_{n=2}^{\infty} \left[ \frac{(\lambda-\alpha)(n-1)+l+n}{l+1} \right]^k z^n$$

where

$$\Psi(z) = \frac{1}{l+1} \left[ \frac{z(\lambda-\alpha+1)}{(1-z)^2} - \frac{z(\lambda-\alpha+l)}{(1-z)} \right]$$

Now

$$\mathcal{I}^k_{\alpha,\lambda,l} = [F(z)]^{-1} * f(z),\ (z \in \mathcal{U}).$$

$$\mathcal{I}^k_{\alpha,\lambda,l} f(z) = z + \sum_{n=2}^{\infty} \left[\frac{l+1}{(\lambda-\alpha)(n-1)+l+n}\right]^k a_n z^n \tag{1.3}$$

where $\alpha \geq 0,\ \lambda \geq 0, n \in N_0, l \geq 0$.

Using the generalized Integral operator in [10] we have defined the following subclasses $\mathcal{L}^k_{\alpha,\lambda,l}(\mu)$, $\mathcal{S}^k_{\alpha,\lambda,l}(\mu)$ as follows

**Definition 1.6.** [10] *A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{L}^k_{\alpha,\lambda,l}(\mu)$ if it satisfy the inequality*

$$\Re\left\{\frac{z(\mathcal{I}^k_{\alpha,\lambda,l} f(z))'}{\mathcal{I}^k_{\alpha,\lambda,l} f(z)}\right\} < \mu, \quad (z \in \mathcal{U})$$

*for some $\mu(\mu > 1)$.*

**Definition 1.7.** [10] *A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{S}^k_{\alpha,\lambda,l}(\mu)$ if it satisfy the inequality*

$$\Re\left\{\frac{z(\mathcal{I}^k_{\alpha,\lambda,l} f(z))''}{(\mathcal{I}^k_{\alpha,\lambda,l} f(z))'}\right\} < \mu, \quad (z \in \mathcal{U})$$

*for some $\mu(\mu > 1)$.*

We have $f \in \mathcal{S}^k_{\alpha,\lambda,l}(\mu)$ if and only if $zf' \in \mathcal{L}^k_{\alpha,\lambda,l}(\mu)$.

**Remark 1.8.** *For $l = 0$ and $l = 0, \alpha = \lambda$ we obtain the integral operator and related subclasses studied by Maslina et.al in* [6] *and Salagean in* [9] *respectively.*

Here we recall the coefficient inequalities associated with the function $f$ belonging to the subclasses $\mathcal{M}^k_{\alpha,\lambda,l}(\mu)$, $\mathcal{N}^k_{\alpha,\lambda,l}(\mu)$, $\mathcal{L}^k_{\alpha,\lambda,l}(\mu)$ and $\mathcal{S}^k_{\alpha,\lambda,l}(\mu)$ respectively .

**Theorem 1.9.** [10] *If the function $f$ belonging to $\mathcal{A}$ satisfies the inequality*

$$\sum_{n=2}^{\infty} \left| \left[\frac{(n-1)(\lambda-\alpha)+l+n}{l+1}\right]^k \right| \{(n-k) + |n+k-2\mu|\}|a_n| \leq 2(\mu-1) \tag{1.4}$$

*for some $0 \leq k \leq 1$ and $\mu > 1$, then $f \in \mathcal{M}^k_{\alpha,\lambda,l}(\mu)$.*

**Theorem 1.10.** [10] *If the function $f$ belonging to $\mathcal{A}$ satisfies the inequality*

$$\sum_{n=2}^{\infty} \left| \left[\frac{(n-1)(\lambda-\alpha)+l+n}{l+1}\right]^k \right| n\{(n-k) + |n+k-2\mu|\}|a_n| \leq 2(\mu-1) \tag{1.5}$$

*for some $0 \leq k \leq 1$ and $\mu > 1$, then $f \in \mathcal{N}^k_{\alpha,\lambda,l}(\mu)$.*

**Theorem 1.11.** [10] *If the function $f$ belonging to $\mathcal{A}$ satisfies the inequality*

$$\sum_{n=2}^{\infty}\left|\left[\frac{(l+1)}{(n-1)(\lambda-\alpha)+l+n}\right]^{k}\right|\{(n-k)+|n+k-2\mu|\}|a_n|\leq 2(\mu-1) \tag{1.6}$$

*for some $0\leq k\leq 1$ and $\mu>1$, then $f\in\mathcal{L}^{k}_{\alpha,\lambda,l}(\mu)$.*

**Theorem 1.12.** [10] *If the function $f$ belonging to $\mathcal{A}$ satisfies the inequality*

$$\sum_{n=2}^{\infty}\left|\left[\frac{(l+1)}{(n-1)(\lambda-\alpha)+l+n}\right]^{k}\right|n\{(n-k)+|n+k-2\mu|\}|a_n|\leq 2(\mu-1) \tag{1.7}$$

*for some $0\leq k\leq 1$ and $\mu>1$, then $f\in\mathcal{S}^{k}_{\alpha,\lambda,l}(\mu)$.*

**Remark 1.13.** *Suitable choices of parameters yield the coefficient inequalities derived by Owa et. al in* [7] *and Maslina Darus et. al in* [6].

## 2. Subordination results

In our present investigation we need the following definition and also a related lemma due to Wilf [14].

**Definition 2.1.** *For two functions $f$ and $g$ analytic in $\mathcal{U}$, we say that the function $f$ is subordinate to $g$ in $\mathcal{U}$ and write $f\prec g$, if there exists a Schwarz function $\omega$, which is analytic in $\mathcal{U}$ with $\omega(0)=0$ and $|\omega(z)|<1$, such that $f(z)=g(\omega(z)),\quad z\in\mathcal{U}$.*

**Definition 2.2.** *(Subordinating factor sequence) A sequence $\{b_n\}_{n=1}^{\infty}$ of complex numbers is said to be a subordinating factor sequence if, whenever $f$ of the form ( 1.1) is analytic, univalent and convex in $\mathcal{U}$, we have the subordination given by*

$$\sum_{n=1}^{\infty}a_nb_nz^n\prec f(z)\quad(z\in\mathcal{U},a_1=1).$$

**Lemma 2.3.** [14] *The sequence $\{b_n\}_{n=1}^{\infty}$ is a subordinating factor sequence if and only if* $\Re\left\{1+2\sum_{n=1}^{\infty}b_nz^n\right\}>0,\quad(z\in\mathcal{U}).$

First we obtain the Subordination results for the functions in the class $\mathcal{M}^{k}_{\alpha,\lambda,l}(\mu)$.

**Theorem 2.4.** *Let $f\in\mathcal{M}^{k}_{\alpha,\lambda,l}(\mu)$ and suppose that $g\in\mathcal{K}$. Then*

$$\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^{k}\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^{k}\right|\{(2-k)+|2(\mu-1)-k|\}\right\}}(f*g)(z)\prec g(z) \tag{2.1}$$

*for every function* $g \in \mathcal{K}$, $\quad z \in \mathcal{U}$, *and*

$$\Re\{f(z)\} > -\frac{2(\mu-1) + \left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}, \quad (z \in \mathcal{U}). \tag{2.2}$$

*The constant factor*

$$\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1) + \left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}} \tag{2.3}$$

*is the best estimate.*

*Proof.* Let $f \in \mathcal{M}^k_{\alpha,\lambda,l}(\mu)$ and suppose that $\quad g(z) = z + \sum_{n=2}^{\infty} c_n z^n \in \mathcal{K}$. Then we have

$$\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1) + \left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}}(f*g)(z)$$

$$= \frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1) + \left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}}(z + \sum_{n=2}^{\infty} a_n c_n z^n)$$

By definition ( 2.2) the subordination result ( 2.1) holds true if the sequence

$$\left\{\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1) + \left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}} a_n\right\}_{n=1}^{\infty} \tag{2.4}$$

is a subordinating factor sequence with $a_1 = 1$.

In view of lemma ( 2.3) it is enough to prove the inequality:

(2.5)

$$\Re\left\{1 + \sum_{n=1}^{\infty} \frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1) + \left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}} a_n z^n\right\} > 0, \quad (z \in \mathcal{U}).$$

Now,

$$\Re\left\{1+\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}\sum_{n=1}^{\infty}a_nz^n\right\}$$

$$=\Re\left\{1+\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}z\right.$$
$$+\frac{1}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}$$
$$\left.\sum_{n=2}^{\infty}\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}a_nz^n\right\}.$$

when $|z|=r,(0<r<1)$,

$$\geq 1-\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}r$$
$$-\frac{1}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}$$
$$\sum_{n=2}^{\infty}\left|\left[\frac{(n-1)(\lambda-\alpha)+l+n}{l+1}\right]^k\right|\{(n-k)+|n+k-2\mu|\}|a_n|r^n$$

$$\geq 1-\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}r$$
$$-\frac{2(\mu-1)}{2(\mu-1)+\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}r$$
$$=1-r>0$$

Then (2.5) holds in $\mathcal{U}$. This proves the inequality (2.1). The inequality (2.2) follows from (2.5), by taking convex function $g(z)=\dfrac{z}{1-z}=z+\displaystyle\sum_{n=2}^{\infty}z^n$. To prove the sharpness of the constant $\dfrac{\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1)+\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}}$ we consider the function $f_0 \in \mathcal{M}^k_{\alpha,\lambda,l}(\mu)$ given by

$$f_0(z) = z - \frac{2(\mu-1)}{\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}} z^2, \tag{2.6}$$

from (2.1),

$$\frac{\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1)+\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}} f_0(z) \prec \frac{z}{1-z}, \quad (z \in \mathcal{U})$$

For the function $f_0$, it is easy to verify that

$$\min\left\{\Re\left\{\frac{\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1)+\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}} f_0(z)\right\}\right\} = -\frac{1}{2}. \, (|z| \le 1)$$

This shows that the constant $\dfrac{\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1)+\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}}$ is the best possible, which completes the proof. $\square$

**Theorem 2.5.** *Let $f \in \mathcal{N}^k_{\alpha,\lambda,l}(\mu)$ and suppose that $g \in \mathcal{K}$. Then*

$$\frac{\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{\left\{2(\mu-1)+2\left|\left[\dfrac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}}(f*g)(z) \prec g(z) \tag{2.7}$$

*for every function $g \in \mathcal{K}$, $\quad z \in \mathcal{U}$, and*

(2.8)

$$\Re\{f(z)\} > -\frac{2(\mu-1) + 2\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{2\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}, \quad (z \in \mathcal{U}).$$

*The constant factor*

$$\frac{\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}}{\left\{2(\mu-1) + 2\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right|\{(2-k)+|2(\mu-1)-k|\}\right\}} \tag{2.9}$$

*is the best estimate.*

**Theorem 2.6.** *Let $f \in \mathcal{L}^k_{\alpha,\lambda,l}(\mu)$ and suppose that $g \in \mathcal{K}$. Then*

$$\frac{(2-k)+|2(\mu-1)-k|}{2\left\{2(\mu-1)\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right| + (2-k)+|2(\mu-1)-k|\right\}}(f*g)(z) \prec g(z) \tag{2.10}$$

*for every function $g \in \mathcal{K}$, $\quad z \in \mathcal{U}$, and*

$$\Re\{f(z)\} > -\frac{2(\mu-1)\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k + \right|\{(2-k)+|2(\mu-1)-k|\}}{\{(2-k)+|2(\mu-1)-k|\}}, \quad (z \in \mathcal{U}). \tag{2.11}$$

*The constant factor*

$$\frac{\{(2-k)+|2(\mu-1)-k|\}}{2\left\{2(\mu-1)\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right| + \{(2-k)+|2(\mu-1)-k|\}\right\}} \tag{2.12}$$

*is the best estimate.*

**Theorem 2.7.** *Let $f \in \mathcal{S}^k_{\alpha,\lambda,l}(\mu)$ and suppose that $g \in \mathcal{K}$. Then*

$$\frac{\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1)\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right| + 2\{(2-k)+|2(\mu-1)-k|\}}(f*g)(z) \prec g(z) \tag{2.13}$$

*for every function $g \in \mathcal{K}$, $\quad z \in \mathcal{U}$, and*
(2.14)

$$\Re\{f(z)\} > -\frac{2(\mu-1)\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right| + 2\{(2-k)+|2(\mu-1)-k|\}}{\{(2-k)+|2(\mu-1)-k|\}}, \quad (z \in \mathcal{U}).$$

*The constant factor*

$$\frac{\{(2-k)+|2(\mu-1)-k|\}}{2(\mu-1)\left|\left[\frac{(2+l+\lambda-\alpha)}{l+1}\right]^k\right| + 2\{(2-k)+|2(\mu-1)-k|\}} \tag{2.15}$$

*is the best estimate.*

**Remark 2.8.** *For $l = 0$ we obtain the subordination results derived in* [8].

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