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NUMERICAL APPROACH FOR SOLVING A SYSTEM OF DUAL FUZZY POLYNOMIAL EQUATIONS

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Abstract. System of fuzzy polynomial equations, play a major role in several applications in various area such as engineering, physics and economics. In this paper, we present numerical approach for solving a system of dual fuzzy polynomial equations based on Newton's method. Also, some numerical examples are given to show the efficiency of algorithms.

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1. Introduction

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh [25], Dubois and Prade [15]. One of the major applications of fuzzy number arithmetic is treating system of fuzzy polynomials, several problems in various areas such as economics, engineering and physics boil down to the solution of a system of fuzzy polynomial equations.

Abbasbandy [3] improved Newton-Raphson method to solve the nonlinear equation $f(x) = 0$ based on modified Adomian's method, and in [4] he extended Newton's method for a system of nonlinear equation by modified Adomian decomposition method.

The concept of fuzzy numbers and arithmetic operation with these numbers were first introduce and investigated by [13, 15, 20]. One of the major applications of fuzzy number arithmetic is in nonlinear systems whose parameters are all or partially represented by fuzzy numbers [14, 17, 19].

Abbasbandy and Asady [5], applied the Newton's method for solving fuzzy nonlinear equations, $f(x) = c$ and the numerical solution of a fuzzy nonlinear equation and system of fuzzy nonlinear equations was considered in [7, 21, 6]. Allahviranloo et al [12] applied the Fixed point method for solving fuzzy nonlinear equations. Tavassoli et al [24], applied the Newton's method for solving dual fuzzy nonlinear equations,

$f(x) = g(x) + c$. The topic of numerical solution of fuzzy polynomials by fuzzy neural network investigated by Abbasbandy *et al.* [8], this method for finding solution to polynomials of the form $a_1x + a_2x^2 + \ldots + a_nx^n = a_0$ for $x \in \mathbb{R}$ (if exists) and $a_0, a_1, \ldots, a_n$ are fuzzy numbers and system of s fuzzy polynomial equations such as [9]:

$$\begin{aligned}
&f_1(x_1, x_2, \ldots, x_n) = a_{10},\\
&\vdots\\
&f_l(x_1, x_2, \ldots, x_n) = a_{l0},\\
&\vdots\\
&f_s(x_1, x_2, \ldots, x_n) = a_{s0},
\end{aligned}$$

where $x_1, x_2, \ldots, x_n \in \mathbb{R}$ and all coefficients are fuzzy numbers. Otadi and Mosleh [23] applied the Adomian decomposition method for solving fuzzy polynomial equation of the form $a_1x + a_2x^2 + \ldots + a_nx^n = a_0$ where x, a_0 and all coefficients are fuzzy numbers. It is the purpose of this paper to introduce an efficient extension of Newton's method by modified Adomian decomposition method for solving (if it exists) system of dual fuzzy polynomials then Mosleh [22] considered dual fuzzy polynomial equation and applied the Adomian decomposition method. In this paper, we consider system of dual fuzzy polynomial equations.

The structure of this paper is organized as follows:
In Section 2, we recall some fundamental results on fuzzy numbers. The proposed algorithm for finding a fuzzy root (if it exists) of a system of fuzzy polynomials are discussed in Section 3. This leads us to conclude by giving a comparison with other methods in Section 4. Numerical examples are given in Section 5.

2. PRELIMINARIES

Definition 1 [18]. A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r); 0 \leq r \leq 1$ which satisfy the following requirements:

i. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function on $(0, 1]$ and right continuous at 0.

ii. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function on $(0, 1]$ and right continuous at 0.

iii. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

The set of all these fuzzy numbers is denoted by E. A popular fuzzy number is the trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta)$ with interval defuzzifier $[x_0, y_0]$ and left fuzziness σ and right fuzziness β where the membership function is

$$u(x) = \begin{cases} \frac{x-x_0+\sigma}{\sigma}, & x_0-\sigma \le x \le x_0, \\ 1 & x \in [x_0, y_0], \\ \frac{y_0-x+\beta}{\beta} & y_0 \le x \le y_0+\beta, \\ 0 & otherwise. \end{cases}$$

Its parametric form is

$$\underline{u}(r) = x_0 - \sigma + \sigma r, \qquad \overline{u}(r) = y_0 + \beta - \beta r. \tag{1}$$

Let $u = (x_0, y_0, \sigma, \beta)$, be a trapezoidal fuzzy number and $x_0 = y_0$, then u is called a triangular fuzzy number and is denoted by $u = (x_0, \delta, \beta)$.

The addition and scaler multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows.

For arbitrary $u = (\underline{u}, \overline{u}), v = (\underline{v}, \overline{v})$ and $k > 0$ we define addition $(u+v)$, multiplication $(u.v)$ and multiplication by scalar k as

$$\begin{gathered} (\underline{u+v})(r) = \underline{u}(r) + \underline{v}(r), \quad (\overline{u+v})(r) = \overline{u}(r) + \overline{v}(r), \\ (\underline{u.v})(r) = min\{\underline{u}(r).\underline{v}(r), \underline{u}(r).\overline{v}(r), \overline{u}(r).\underline{v}(r), \overline{u}(r).\overline{v}(r)\}, \\ (\overline{u.v})(r) = max\{\underline{u}(r).\underline{v}(r), \underline{u}(r).\overline{v}(r), \overline{u}(r).\underline{v}(r), \overline{u}(r).\overline{v}(r)\}, \\ (\underline{ku})(r) = k\underline{u}(r), \qquad (\overline{ku})(r) = k\overline{u}(r). \end{gathered} \tag{2}$$

Definition 3. Let u and v be fuzzy numbers with r-level set $[u]^r = [u_1(r), u_2(r)]$ and $[v]^r = [v_1(r), v_2(r)]$. We metricize the set of fuzzy numbers by the Hausdorff distance

$$D(u, v) = sup_{r \in [0,1]} max\{\mid u_1(r) - v_1(r) \mid, \mid u_2(r) - v_2(r) \mid\}. \tag{3}$$

i.e. $D(u, v)$ is the maximal distance between r level sets of u and v.

3. SYSTEM OF DUAL FUZZY POLYNOMIAL EQUATIONS

Usually, there is no inverse element for an arbitrary fuzzy number $u \in E$, i.e., there exists no element $v \in E$ such that

$$u + v = 0.$$

Actually, for all non-crisp fuzzy number $u \in E^1$ we have

$$u + (-u) \neq 0.$$

Therefore, the system of fuzzy polynomial equations

$$\begin{cases} P_1(x_1, x_2, \ldots, x_n) = Q_1(x_1, x_2, \ldots, x_n) + c_1, \\ \vdots \\ P_l(x_1, x_2, \ldots, x_n) = Q_l(x_1, x_2, \ldots, x_n) + c_l, \\ \vdots \\ P_s(x_1, x_2, \ldots, x_n) = Q_s(x_1, x_2, \ldots, x_n) + c_s, \end{cases} \tag{4}$$

with

$$P_l(x_1, x_2, \ldots, x_n) = c_l = \sum_{i=1}^n a_{li} x_i + \sum_{i=1}^n \sum_{j=1}^n a_{lij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{lijk} x_i x_j x_k + \ldots$$

and

$$Q_l(x_1, x_2, \ldots, x_n) = c_l = \sum_{i=1}^n b_{li} x_i + \sum_{i=1}^n \sum_{j=1}^n b_{lij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n b_{lijk} x_i x_j x_k + \ldots, \quad 1 \leq l \leq s,$$

where $x_1, x_2, \ldots, x_n$ and all coefficients are fuzzy numbers, cannot be equivalently replaced by the system of fuzzy polynomial equations

$$\begin{cases} P_1(x_1, x_2, \ldots, x_n) - Q_1(x_1, x_2, \ldots, x_n) = c_1, \\ \vdots \\ P_l(x_1, x_2, \ldots, x_n) - Q_l(x_1, x_2, \ldots, x_n) = c_l, \\ \vdots \\ P_s(x_1, x_2, \ldots, x_n) - Q_s(x_1, x_2, \ldots, x_n) = c_s, \end{cases} \tag{5}$$

which had been investigated. In the sequel, we will call the system of fuzzy polynomial equations (4), system of dual fuzzy polynomial equations.

This full form of mathematical description can be represented by a system of partial quadratic fuzzy polynomials consisting of only two variables in the form of

$$\begin{cases} P_1(x, y) = a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2, \\ Q_1(x, y) = b_1 x + b_2 y + b_3 xy + b_4 x^2 + b_5 y^2, \\ P_2(x, y) = h_1 x + h_2 y + h_3 xy + h_4 x^2 + h_5 y^2, \\ Q_2(x, y) = d_1 x + d_2 y + d_3 xy + d_4 x^2 + d_5 y^2, \end{cases} \tag{6}$$

where x, y, c_1, c_2 and all coefficients are fuzzy numbers.
Let

$$\begin{cases} P_1(x,y) = (\underline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r), \overline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r)), \\ Q_1(x,y) = (\underline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r), \overline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r)), \\ P_2(x,y) = (\underline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r), \overline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r)), \\ Q_2(x,y) = (\underline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r), \overline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r)), \quad for \ \ r \in [0,1], \end{cases}$$

with

$$\begin{cases} \underline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) = min\{P_1(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], a_i \in [\underline{a}_i(r),\overline{a}_i(r)], i = 1,...,5\}, \\ \overline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) = max\{P_1(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], a_i \in [\underline{a}_i(r),\overline{a}_i(r)], i = 1,...,5\}, \\ \underline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) = min\{Q_1(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], b_i \in [\underline{b}_i(r),\overline{b}_i(r)], i = 1,...,5\}, \\ \overline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) = max\{Q_1(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], b_i \in [\underline{b}_i(r),\overline{b}_i(r)], i = 1,...,5\} \\ \underline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) = min\{P_2(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], h_i \in [\underline{h}_i(r),\overline{h}_i(r)], i = 1,...,5\}, \\ \overline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) = max\{P_2(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], h_i \in [\underline{h}_i(r),\overline{h}_i(r)], i = 1,...,5\}, \\ \underline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) = min\{Q_2(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], d_i \in [\underline{d}_i(r),\overline{d}_i(r)], i = 1,...,5\}, \\ \overline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) = max\{Q_2(u,v) \mid u \in [\underline{x}(r),\overline{x}(r)], \\ \qquad\qquad v \in [\underline{y}(r),\overline{y}(r)], d_i \in [\underline{d}_i(r),\overline{d}_i(r)], i = 1,...,5\}. \end{cases}$$

The parametric form for any $r \in [0,1]$, is as follows:

$$\begin{cases} \underline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \underline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) + \underline{c}_1(r), \\ \overline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \overline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) + \overline{c}_1(r), \\ \underline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \underline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) + \underline{c}_2(r), \\ \overline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \overline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) + \overline{c}_2(r), \end{cases} \tag{7}$$

where $c_1 = (\underline{c}_1(r), \overline{c}_1(r))$ and $c_2 = (\underline{c}_2(r), \overline{c}_2(r))$. The problem (7) can be reformulated in an equivalent form as

$$\begin{cases} \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) = 0, \\ \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) = 0, \\ \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) = 0, \\ \overline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) = 0, \end{cases} \tag{8}$$

where

$$\begin{cases} \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \underline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \underline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \underline{c}_1(r), \\ \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \overline{P_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \overline{Q_1}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \overline{c}_1(r), \\ \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \underline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \underline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \underline{c}_2(r), \\ \overline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) = \overline{P_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \overline{Q_2}(\underline{x},\overline{x},\underline{y},\overline{y};r) - \overline{c}_2(r). \end{cases}$$

Suppose that $(\alpha,\beta,\gamma,\theta)$ is the solution of (8), i.e.,

$$\begin{cases} \underline{F}(\alpha,\beta,\gamma,\theta;r) = 0, \\ \overline{F}(\alpha,\beta,\gamma,\theta;r) = 0, \\ \underline{G}(\alpha,\beta,\gamma,\theta;r) = 0, \\ \overline{G}(\alpha,\beta,\gamma,\theta;r) = 0. \end{cases}$$

Now if we use the Taylor series of $\underline{F},\overline{F},\underline{G},\overline{G}$ about $(\underline{x},\overline{x},\underline{y},\overline{y})$, then for each $r \in [0,1]$,

$$\begin{cases} \underline{F}(\underline{x}-h,\overline{x}-k,\underline{y}-l,\overline{y}-d;r) = \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\underline{F}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -k\underline{F}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - l\underline{F}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\underline{F}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \\ \overline{F}(\underline{x}-h,\overline{x}-k,\underline{y}-l,\overline{y}-d;r) = \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\overline{F}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -k\overline{F}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - l\overline{F}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\overline{F}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \\ \underline{G}(\underline{x}-h,\overline{x}-k,\underline{y}-l,\overline{y}-d;r) = \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\underline{G}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -k\underline{G}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - l\underline{G}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\underline{G}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \\ \overline{G}(\underline{x}-h,\overline{x}-k,\underline{y}-l,\overline{y}-d;r) = \overline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\overline{G}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -k\overline{G}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - l\overline{G}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\overline{G}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \end{cases}$$

that $\underline{F}_{\underline{x}}$ means that, the derivative of $\underline{F}$ with respect to $\underline{x}$ and so on. We assume, of course, that all needed partial derivatives exist and are bounded. Therefore for sufficiently small $h(r), k(r), l(r)$ and $d(r)$ for each $r \in [0,1]$,

$$\begin{cases} \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\underline{F}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - k\underline{F}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -l\underline{F}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\underline{F}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \simeq 0, \\ \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\overline{F}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - k\overline{F}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -l\overline{F}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\overline{F}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \simeq 0, \\ \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\underline{G}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - k\underline{G}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -l\underline{G}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\underline{G}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \simeq 0, \\ \overline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) - h\overline{G}_{\underline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - k\overline{G}_{\overline{x}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \qquad -l\overline{G}_{\underline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) - d\overline{G}_{\overline{y}}(\underline{x},\overline{x},\underline{y},\overline{y};r) \simeq 0, \end{cases}$$

and hence $h(r), k(r), l(r)$ and $d(r)$ are unknown quantities that can be obtained by solving the following equations, for each $r \in [0,1]$

$$\text{(9)} \qquad J(\underline{x},\overline{x},\underline{y},\overline{y};r)\begin{bmatrix} h(r) \\ k(r) \\ l(r) \\ d(r) \end{bmatrix} = \begin{bmatrix} \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r) \\ \overline{G}(\underline{x},\overline{x},,\underline{y},\overline{y};r) \end{bmatrix},$$

where

$$J(\underline{x},\overline{x},\underline{y},\overline{y};r) = \begin{bmatrix} \underline{F}_{\underline{x}}\ \underline{F}_{\overline{x}}\ \underline{F}_{\underline{y}}\ \underline{F}_{\overline{y}} \\ \overline{F}_{\underline{x}}\ \overline{F}_{\overline{x}}\ \overline{F}_{\underline{y}}\ \overline{F}_{\overline{y}} \\ \underline{G}_{\underline{x}}\ \underline{G}_{\overline{x}}\ \underline{G}_{\underline{y}}\ \underline{F}_{\overline{y}} \\ \overline{G}_{\underline{x}}\ \overline{G}_{\overline{x}}\ \overline{G}_{\underline{y}}\ \overline{F}_{\overline{y}} \end{bmatrix}(\underline{x},\overline{x},\underline{y},\overline{y};r).$$

The Newton's method is given by

$$\text{(10)} \qquad \begin{cases} \underline{x}_{n+1}(r) = \underline{x}_n(r) + h_n(r), \\ \overline{x}_{n+1}(r) = \overline{x}_n(r) + k_n(r), \\ \underline{y}_{n+1}(r) = \underline{y}_n(r) + l_n(r), \\ \overline{y}_{n+1}(r) = \overline{y}_n(r) + d_n(r), \end{cases}$$

where $n = 0, 1, 2, \ldots$ and $h_n(r), k_n(r), l_n(r), d_n(r)$ are given by (9). For initial guess, one can use the trapezoidal fuzzy number

$$x_0 = (\underline{x}(1), \overline{x}(1), \underline{x}(1) - \underline{x}(0), \overline{x}(0) - \overline{x}(1)),$$

$$y_0 = (\underline{y}(1), \overline{y}(1), \underline{y}(1) - \underline{y}(0), \overline{y}(0) - \overline{y}(1)),$$

and in parametric form

$$\begin{aligned} \underline{x}_0(r) &= \underline{x}(1) + (\underline{x}(1) - \underline{x}(0))(r-1), \\ \overline{x}_0(r) &= \overline{x}(1) + (\overline{x}(0) - \overline{x}(1))(1-r), \\ \underline{y}_0(r) &= \underline{y}(1) + (\underline{y}(1) - \underline{y}(0))(r-1), \\ \overline{y}_0(r) &= \overline{y}(1) + (\overline{y}(0) - \overline{y}(1))(1-r). \end{aligned}$$

The iteration (10) will converge to $(\alpha, \beta, \gamma, \theta)$ if the starting point $(\underline{x}_0(r), \overline{x}_0(r)\ , \underline{y}_0(r), \overline{y}_0(r))$ is close enough to $(\alpha, \beta, \gamma, \theta)$ for $0 \leq r \leq 1$, local convergence property, see [11] for more details.

If we use Taylor's expansion of $\underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)$ and $\overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)$ to a higher order and we are looking for $h(r), k(r), l(r)$ and $d(r)$ such as:

$$[\underline{F} - h\underline{F}_{\underline{x}} - k\underline{F}_{\overline{x}} - l\underline{F}_{\underline{y}} - d\underline{F}_{\overline{y}} + \frac{1}{2}(h^2\underline{F}_{\underline{x}\,\underline{x}} + k^2\underline{F}_{\overline{x}\,\overline{x}} + l^2\underline{F}_{\underline{y}\,\underline{y}} + d^2\underline{F}_{\overline{y}\,\overline{y}}$$

$$+2hk\underline{F}_{\underline{x}\,\overline{x}}+2ld\underline{F}_{\underline{y}\,\overline{y}}+2hl\underline{F}_{\underline{x}\,\underline{y}}++2hd\underline{F}_{\underline{x}\,\overline{y}}+2kl\underline{F}_{\overline{x}\,\underline{y}}++2kd\underline{F}_{\overline{x}\,\overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

$$[\overline{F}-h\overline{F}_{\underline{x}}-k\overline{F}_{\overline{x}}-l\overline{F}_{\underline{y}}-d\overline{F}_{\overline{y}}+\frac{1}{2}(h^2\overline{F}_{\underline{x}\,\underline{x}}+k^2\overline{F}_{\overline{x}\,\overline{x}}+l^2\overline{F}_{\underline{y}\,\underline{y}}+d^2\overline{F}_{\overline{y}\,\overline{y}}$$

$$+2hk\overline{F}_{\underline{x}\,\overline{x}}+2ld\overline{F}_{\underline{y}\,\overline{y}}+2hl\overline{F}_{\underline{x}\,\underline{y}}++2hd\overline{F}_{\underline{x}\,\overline{y}}+2kl\overline{F}_{\overline{x}\,\underline{y}}++2kd\overline{F}_{\overline{x}\,\overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

$$[\underline{G}-h\underline{G}_{\underline{x}}-k\underline{G}_{\overline{x}}-l\underline{G}_{\underline{y}}-d\underline{G}_{\overline{y}}+\frac{1}{2}(h^2\underline{G}_{\underline{x}\,\underline{x}}+k^2\underline{G}_{\overline{x}\,\overline{x}}+l^2\underline{G}_{\underline{y}\,\underline{y}}+d^2\underline{G}_{\overline{y}\,\overline{y}}$$

$$+2hk\underline{G}_{\underline{x}\,\overline{x}}+2ld\underline{G}_{\underline{y}\,\overline{y}}+2hl\underline{G}_{\underline{x}\,\underline{y}}++2hd\underline{G}_{\underline{x}\,\overline{y}}+2kl\underline{G}_{\overline{x}\,\underline{y}}++2kd\underline{G}_{\overline{x}\,\overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

$$[\overline{G}-h\overline{G}_{\underline{x}}-k\overline{G}_{\overline{x}}-l\overline{G}_{\underline{y}}-d\overline{G}_{\overline{y}}+\frac{1}{2}(h^2\overline{G}_{\underline{x}\,\underline{x}}+k^2\overline{G}_{\overline{x}\,\overline{x}}+l^2\overline{G}_{\underline{y}\,\underline{y}}+d^2\overline{G}_{\overline{y}\,\overline{y}}$$

$$+2hk\overline{G}_{\underline{x}\,\overline{x}}+2ld\overline{G}_{\underline{y}\,\overline{y}}+2hl\overline{G}_{\underline{x}\,\underline{y}}++2hd\overline{G}_{\underline{x}\,\overline{y}}+2kl\overline{G}_{\overline{x}\,\underline{y}}++2kd\overline{G}_{\overline{x}\,\overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

given

$$h(r)=[\underline{F}-k\underline{F}_{\overline{x}}-l\underline{F}_{\underline{y}}-d\underline{F}_{\overline{y}}+\frac{1}{2}(h^2\underline{F}_{\underline{x}\,\underline{x}}+k^2\underline{F}_{\overline{x}\,\overline{x}}+l^2\underline{F}_{\underline{y}\,\underline{y}}+d^2\underline{F}_{\overline{y}\,\overline{y}}+2hk\underline{F}_{\underline{x}\,\overline{x}}$$

$$+2ld\underline{F}_{\underline{y}\,\overline{y}}+2hl\underline{F}_{\underline{x}\,\underline{y}}++2hd\underline{F}_{\underline{x}\,\overline{y}}+2kl\underline{F}_{\overline{x}\,\underline{y}}++2kd\underline{F}_{\overline{x}\,\overline{y}})/\underline{F}_{\underline{x}}](\underline{x},\overline{x},\underline{y},\overline{y};r),$$

$$k(r)=[\overline{F}-h\overline{F}_{\underline{x}}-l\overline{F}_{\underline{y}}-d\overline{F}_{\overline{y}}+\frac{1}{2}(h^2\overline{F}_{\underline{x}\,\underline{x}}+k^2\overline{F}_{\overline{x}\,\overline{x}}+l^2\overline{F}_{\underline{y}\,\underline{y}}+d^2\overline{F}_{\overline{y}\,\overline{y}}+2hk\overline{F}_{\underline{x}\,\overline{x}}$$

$$+2ld\overline{F}_{\underline{y}\,\overline{y}}+2hl\overline{F}_{\underline{x}\,\underline{y}}++2hd\overline{F}_{\underline{x}\,\overline{y}}+2kl\overline{F}_{\overline{x}\,\underline{y}}++2kd\overline{F}_{\overline{x}\,\overline{y}})/\overline{F}_{\overline{x}}](\underline{x},\overline{x},\underline{y},\overline{y};r),$$

$$l(r)=[\underline{G}-h\underline{G}_{\underline{x}}-k\underline{G}_{\overline{x}}-d\underline{G}_{\overline{y}}+\frac{1}{2}(h^2\underline{G}_{\underline{x}\,\underline{x}}+k^2\underline{G}_{\overline{x}\,\overline{x}}+l^2\underline{G}_{\underline{y}\,\underline{y}}+d^2\underline{G}_{\overline{y}\,\overline{y}}+2hk\underline{G}_{\underline{x}\,\overline{x}}$$

$$+2ld\underline{G}_{\underline{y}\,\overline{y}}+2hl\underline{G}_{\underline{x}\,\underline{y}}++2hd\underline{G}_{\underline{x}\,\overline{y}}+2kl\underline{G}_{\overline{x}\,\underline{y}}++2kd\underline{G}_{\overline{x}\,\overline{y}})/\underline{G}_{\underline{y}}](\underline{x},\overline{x},\underline{y},\overline{y};r),$$

$$d(r)=[\overline{G}-h\overline{G}_{\underline{x}}-k\overline{G}_{\overline{x}}-l\overline{G}_{\underline{y}}+\frac{1}{2}(h^2\overline{G}_{\underline{x}\,\underline{x}}+k^2\overline{G}_{\overline{x}\,\overline{x}}+l^2\overline{G}_{\underline{y}\,\underline{y}}+d^2\overline{G}_{\overline{y}\,\overline{y}}+2hk\overline{G}_{\underline{x}\,\overline{x}}$$

$$+2ld\overline{G}_{\underline{y}\,\overline{y}}+2hl\overline{G}_{\underline{x}\,\underline{y}}++2hd\overline{G}_{\underline{x}\,\overline{y}}+2kl\overline{G}_{\overline{x}\,\underline{y}}++2kd\overline{G}_{\overline{x}\,\overline{y}})/\overline{G}_{\overline{y}}](\underline{x},\overline{x},\underline{y},\overline{y};r),$$

or

$$\begin{bmatrix} h(r)\\ k(r)\\ l(r)\\ d(r)\end{bmatrix}=\begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4\end{bmatrix}+N\left(\begin{bmatrix} h(r)\\ k(r)\\ l(r)\\ d(r)\end{bmatrix}\right)=\begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4\end{bmatrix}+\begin{bmatrix} N_1(h,k,l,d)\\ N_2(h,k,l,d)\\ N_3(h,k,l,d)\\ N_4(h,k,l,d)\end{bmatrix}, \tag{11}$$

where $e_1=\frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x},\overline{x},\underline{y},\overline{y};r)$, $e_2=\frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x},\overline{x},\underline{y},\overline{y};r)$, $e_3=\frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x},\overline{x},\underline{y},\overline{y};r)$ and $e_4=\frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x},\overline{x},\underline{y},\overline{y};r)$ are constants and N is a vector quadratic polynomial and for approximating $h(r), k(r), l(r)$ and $d(r)$, we can apply the multivariable Adomian decomposition method [1].

The Adomian decomposition technique considers representing the solution of (11) as a series

$$h=\sum_{n=0}^{\infty}h_n,\quad k=\sum_{n=0}^{\infty}k_n,\quad l=\sum_{n=0}^{\infty}l_n,\quad d=\sum_{n=0}^{\infty}d_n \tag{12}$$

and the nonlinear functions are decomposed as

$$N_i(h,k,l,d)=\sum_{n=0}^{\infty}A_{in}(h_0,\ldots,h_n,k_0,\ldots,k_n,l_0,\ldots,l_n,,d_0,\ldots,d_n),\quad i=1,...,4. \tag{13}$$

where the A_{in}'s are Adomian's polynomials given by [3],

$$A_{in}=\frac{1}{n!}\frac{d^n}{d\lambda^n}[N_i(\sum_{j=0}^{\infty}\lambda^j h_j,\sum_{j=0}^{\infty}\lambda^j k_j,\sum_{j=0}^{\infty}\lambda^j l_j,\sum_{j=0}^{\infty}\lambda^j d_j)]_{\lambda=0}$$

for $i=1,...,4, j=0,1,\ldots.$

Upon substituting (12), (13) in the (11) yields

$$h_0=e_1,\quad h_{n+1}=A_{1n},\quad k_0=e_2,\quad k_{n+1}=A_{2n},$$

$$l_0=e_3,\quad l_{n+1}=A_{3n},\quad d_0=e_4,\quad d_{n+1}=A_{4n},$$

for $n=0,1,\ldots,$ multivariable polynomials A_{in} are generated by practical formulae presented in [1], for $i=1,2,3,4,$ we have

$$\begin{aligned} A_{i0} &= N_i(h_0,k_0,l_0,d_0),\\ A_{in} &= \textstyle\sum_{\varphi}\frac{h_1^{p_1}}{p_1!}\cdots\frac{h_n^{p_n}}{p_n!}\cdot\frac{k_1^{q_1}}{q_1!}\cdots\frac{k_n^{q_n}}{q_n!}\cdot\frac{l_1^{s_1}}{s_1!}\cdots\frac{l_n^{s_n}}{s_n!}\cdot\frac{d_1^{t_1}}{t_1!}\cdots\frac{d_n^{t_n}}{t_n!}\\ &\quad\cdot\frac{\partial^{\varphi_1+\varphi_2+\varphi_3+\varphi_4}}{\partial h^{\varphi_1}\partial k^{\varphi_2}\partial d^{\varphi_3}\partial l^{\varphi_4}}N_i(h_0,k_0,d_0,l_0),\quad n\neq 0, \end{aligned}$$

where φ stands for $(p_1+2p_2+\ldots+np_n)+(q_1+2q_2+\ldots+nq_n)+(s_1+2s_2+\ldots+ns_n)+(t_1+2t_2+\ldots+nt_n)=n,$ and $\varphi_1=p_1+p_2+\ldots+p_n$, $\varphi_2=q_1+q_2+\ldots+q_n$, $\varphi_3=s_1+s_2+\ldots+s_n$, $\varphi_4=t_1+t_2+\ldots+t_n$.

In practice, of course, the sum of the infinite series has to be truncated at some finite order $M.$ The quantities $\sum_{n=0}^{M}h_n$, $\sum_{n=0}^{M}k_n$, $\sum_{n=0}^{M}l_n$ and $\sum_{n=0}^{M}d_n$, can thus be reasonable approximations of the exact solution of (4), provided M is sufficiently large. As $M\longrightarrow\infty$, the series converge smoothly toward the exact solution for $0\leq r\leq 1$ [2].

Let

$$\begin{aligned} H_M &= h_0+h_1+\ldots+h_M=h_0+A_{10}+A_{11}+\ldots+A_{1M-1},\\ K_M &= k_0+k_1+\ldots+k_M=k_0+A_{20}+A_{21}+\ldots+A_{2M-1},\\ L_M &= l_0+l_1+\ldots+l_M=l_0+A_{30}+A_{31}+\ldots+A_{3M-1},\\ D_M &= d_0+d_1+\ldots+d_M=d_0+A_{40}+A_{41}+\ldots+A_{4M-1}, \end{aligned} \tag{14}$$

denote the $(M+1)$-term approximations of h,k,l and d, respectively. Since the series converge very rapidly, then (14) can serve as a practical solution in each iteration.

We will show that the number of terms required to obtain an accurate computable solution is very small.

Case 1: For $M = 0$

$$h \simeq H_0 = h_0 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$k \simeq K_0 = k_0 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$l \simeq L_0 = l_0 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$d \simeq D_0 = d_0 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\alpha = \underline{x} - h \simeq \underline{x} - H_0 = \underline{x} - \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\beta = \overline{x} - k \simeq \overline{x} - K_0 = \overline{x} - \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\gamma = \underline{y} - l \simeq \underline{y} - L_0 = \underline{y} - \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\theta = \overline{y} - d \simeq \overline{y} - D_0 = \overline{y} - \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$$

and

$$\begin{cases} \underline{x}_{n+1} = \underline{x}_n - \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \\ \\ \overline{x}_{n+1} = \overline{x}_n - \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \\ \\ \underline{y}_{n+1} = \underline{y}_n - \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \\ \\ \overline{y}_{n+1} = \overline{y}_n - \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \end{cases}$$

for $n = 0, 1, \ldots$.

Case 2: For $M = 1$

$$h_1 = A_{1,0} = N_1(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\underline{F}_{\underline{x}\,\underline{x}} + \tfrac{k_0^2}{2}\underline{F}_{\overline{x}\,\overline{x}} + \tfrac{l_0^2}{2}\underline{F}_{\underline{y}\,\underline{y}} + \tfrac{d_0^2}{2}\underline{F}_{\overline{y}\,\overline{y}}$$

$$+h_0k_0\underline{F}_{\underline{x}\,\overline{x}} + h_0l_0\underline{F}_{\underline{x}\,\underline{y}} + h_0d_0\underline{F}_{\underline{x}\,\overline{y}} + k_0l_0\underline{F}_{\overline{x}\,\underline{y}} + k_0d_0\underline{F}_{\underline{x}\,\overline{y}}$$

$$+l_0d_0\underline{F}_{\underline{y}\,\overline{y}})/\underline{F}_{\underline{x}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$k_1 = A_{2,0} = N_2(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\overline{F}_{\underline{x}\ \underline{x}} + \tfrac{k_0^2}{2}\overline{F}_{\overline{x}\ \overline{x}} + \tfrac{l_0^2}{2}\overline{F}_{\underline{y}\ \underline{y}} + \tfrac{d_0^2}{2}\overline{F}_{\overline{y}\ \overline{y}}$$

$$+h_0k_0\overline{F}_{\underline{x}\ \overline{x}} + h_0l_0\overline{F}_{\underline{x}\ \underline{y}} + h_0d_0\overline{F}_{\underline{x}\ \overline{y}} + k_0l_0\overline{F}_{\overline{x}\ \underline{y}} + k_0d_0\overline{F}_{\underline{x}\ \overline{y}}$$

$$+l_0d_0\overline{F}_{\underline{y}\ \overline{y}})/\overline{F}_{\overline{x}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$l_1 = A_{3,0} = N_3(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\underline{G}_{\underline{x}\ \underline{x}} + \tfrac{k_0^2}{2}\underline{G}_{\overline{x}\ \overline{x}} + \tfrac{l_0^2}{2}\underline{G}_{\underline{y}\ \underline{y}} + \tfrac{d_0^2}{2}\underline{G}_{\overline{y}\ \overline{y}}$$

$$+h_0k_0\underline{G}_{\underline{x}\ \overline{x}} + h_0l_0\underline{G}_{\underline{x}\ \underline{y}} + h_0d_0\underline{G}_{\underline{x}\ \overline{y}} + k_0l_0\underline{G}_{\overline{x}\ \underline{y}} + k_0d_0\underline{G}_{\underline{x}\ \overline{y}}$$

$$+l_0d_0\underline{G}_{\underline{y}\ \overline{y}})/\underline{G}_{\underline{y}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$d_1 = A_{4,0} = N_4(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\overline{G}_{\underline{x}\ \underline{x}} + \tfrac{k_0^2}{2}\overline{G}_{\overline{x}\ \overline{x}} + \tfrac{l_0^2}{2}\overline{G}_{\underline{y}\ \underline{y}} + \tfrac{d_0^2}{2}\overline{G}_{\overline{y}\ \overline{y}}$$

$$+h_0k_0\overline{G}_{\underline{x}\ \overline{x}} + h_0l_0\overline{G}_{\underline{x}\ \underline{y}} + h_0d_0\overline{F}_{\underline{x}\ \overline{y}} + k_0l_0\overline{G}_{\overline{x}\ \underline{y}} + k_0d_0\overline{G}_{\underline{x}\ \overline{y}}$$

$$+l_0d_0\overline{G}_{\underline{y}\ \overline{y}})/\overline{G}_{\overline{y}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

where $h_0 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r), k_0 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r), l_0 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$ and $d_0 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$, then

$$\alpha = \underline{x} - h \simeq \underline{x} - H_1 = \underline{x} - h_0 - A_{1,0},$$

$$\beta = \overline{x} - k \simeq \overline{x} - K_1 = \overline{x} - k_0 - A_{2,0},$$

$$\gamma = \underline{y} - l \simeq \underline{y} - L_1 = \underline{y} - l_0 - A_{3,0},$$

$$\theta = \overline{y} - d \simeq \overline{y} - D_1 = \overline{y} - d_0 - A_{4,0}$$

and hence, we have the following iterations:

$$\underline{x}_{n+1} = \underline{x}_n - H_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$

$$\overline{x}_{n+1} = \overline{x}_n - K_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$

$$\underline{y}_{n+1} = \underline{y}_n - L_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$

$$\overline{y}_{n+1} = \overline{y}_n - D_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$

for $n = 0, 1, \ldots$.
We can also obtain similar relations for $M = 2, 3, \ldots$.

The Adomian decomposition method is simply generalized to more variables and upper degrees as well.

4. NUMERICAL EXAMPLES

We consider some examples for the Adomian decomposition method.
In the computer simulation of this examples, we use the following specifications of the Adomian decomposition method.
For each fuzzy numbers, we use $r = 0, 0.1, \ldots, 1$, where we calculate the total error of each iteration by

$$e_i = max\{D(x_i, x_{i-1}), D(y_i, y_{i-1})\}.$$

Example 1. Consider the system of fuzzy polynomial equations

$$\begin{cases} 2x^2 + 2y = x^2 + y + (3, 1, 1.75), \\ 2x + 2y^2 = x + y^2 + (5, 1.4375, 2.75), \end{cases}$$

assume that x and y are positive, then the parametric form of this equation is as follows:

$$\begin{cases} 2\underline{x}^2(r) + 2\underline{y}(r) = \underline{x}^2(r) + \underline{y}(r) + 2 + r, \\ 2\overline{x}^2(r) + 2\overline{y}(r) = \overline{x}^2(r) + \overline{y}(r) + 4.75 - 1.75r, \\ 2\underline{x}(r) + 2\underline{y}^2(r) = \underline{x}(r) + \underline{y}^2(r) + 3.5625 + 1.4375r, \\ 2\overline{x}(r) + 2\overline{y}^2(r) = \overline{x}(r) + \overline{y}^2(r) + 7.75 - 2.75r. \end{cases}$$

Initial guess is $x_0 = (1.25, 0.5, 0.25)$ and $y_0 = (1.75, 0.25, 0.5)$.
For $M = 0$

$$h \simeq H_0 = h_0 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r), \quad k \simeq K_0 = k_0 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$l \simeq L_0 = l_0 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r), \quad d \simeq D_0 = d_0 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\alpha = \underline{x} - h \simeq \underline{x} - H_0 = \underline{x} - \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\beta = \overline{x} - k \simeq \overline{x} - K_0 = \overline{x} - \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\gamma = \underline{y} - l \simeq \underline{y} - L_0 = \underline{y} - \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\theta = \overline{y} - d \simeq \overline{y} - D_0 = \overline{y} - \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

then

$$\begin{cases} \underline{x}_{n+1} = \underline{x}_n - \frac{\underline{x}_n^2 + \underline{y}_n - (2+r)}{2\underline{x}_n}, \\ \overline{x}_{n+1} = \overline{x}_n - \frac{\overline{x}_n^2 + \overline{y}_n - (4.75-1.75r)}{2\overline{x}_n}, \\ \underline{y}_{n+1} = \underline{y}_n - \frac{\underline{x}_n + \underline{y}_n^2 - (3.5625+1.4375r)}{2\underline{y}_n}, \\ \overline{y}_{n+1} = \overline{y}_n - \frac{\overline{x}_n + \overline{y}_n^2 - (7.75-2.75r)}{2\overline{y}_n}, \end{cases}$$

for $n = 0, 1, \ldots, 6$.
By Adomian decomposition method, we obtain the numerical results for $M = 0, 1$. See figures 1,2 and table 1 for more details.

Example 2. Consider the system of fuzzy polynomial equations

$$\begin{cases} 3x^3 + y = 2x^3 + (2.5, 1.375, 4.859375), \\ 2x + 2y^2 = x + y^2 + (3.25, 1.75, 2.5), \end{cases}$$

assume that x and y are positive, then the parametric form of this equation is as follows:

$$\begin{cases} 3\underline{x}^3(r) + \underline{y}(r) = \underline{x}^3(r) + 1.125 + 1.375r, \\ 3\overline{x}^3(r) + \overline{y}(r) = \overline{x}^3(r) + 7.359375 - 4.859375r, \\ 2\underline{x}(r) + 2\underline{y}^2(r) = \underline{x}(r) + \underline{y}^2(r) + 1.5 + 1.75r, \\ 2\overline{x}(r) + 2\overline{y}^2(r) = \overline{x}(r) + \overline{y}^2(r) + 5.75 - 2.5r. \end{cases}$$

Initial guess is $x_0 = (0.75, 0.25, 0.25)$ and $y_0 = (1.25, 0.25, 0.75)$.
By Adomian decomposition method, we obtain the numerical results for $M = 0, 1$. See figures 3,4 and table 2 for more details.

5. Conclusion

In this paper, we proposed numerical method for solving a system of fuzzy nonlinear equations. Initially we wrote fuzzy nonlinear in a parametric form and then solve it by Adomian decomposition method.

REFERENCES

[1] K. Abbaoui, Y. Cherruault, V. Seng, Practical formulae for the calculuse of multivariable Adomian polynomials, Comput. Model. 22 (1995) 89-93.

[2] K. Abbaoui, Y. Cherruault, Convergence of Adomian's method applied to nonlinear equations, Math. Comput. Model 20 (9)(1994) 69-73.

[3] S. Abbasbandy, Improving Newton- Raphson method for nonlinear equations by modified Adomian decomposition method. Appl. Math. Comput. 145 (2003) 887-893.

[4] S. Abbasbandy, Extended Newton's method for a system of nonlinear equations by modified Adomian decomposition method. Appl. Math. Comput. 170 (2005) 648-656.

[5] S. Abbasbandy, and B. Asady, Newtons method for solving fuzzy nonlinear equations. Appl. Math. Comput. 159 (2004), 349-356.

[6] S. Abbasbandy, R. Ezzati, Newton's method for solving a system of fuzzy nonlinear equations, Appl. Math. Comput. 175 (2006) 1189-1199.

[7] S. Abbasbandy, and A. Jafarian, Steepest descent method for solving fuzzy nonlinear equations. Appl. Math. Comput. 175 (2006) 823-833.

[8] S. Abbasbandy and M. Otadi, Numerical solution of fuzzy polynomials by fuzzy neural network, Appl. Math. Comput. 181 (2006) 1084-1089.

[9] S. Abbasbandy, M. Otadi and M. Mosleh, Numerical solution of a system of fuzzy polynomials by fuzzy neural network, In form. Sci. 178 (2008) 1948-1960 .

[10] G. Adomian, Nonlinear stochastic system and approximations to physics, klower academic publisher, Dordrecht, 1989.

[11] G. Adomian, R. Rach, On the solution of algebraic equations by yhe decomposition method, Math. Anal. Appl 105 (1985) 141-166.

[12] T. Allahviranloo, M. Otadi and M. Mosleh, Iterative method for fuzzy equations, Soft Comput,12 (2007) 935-939.

[13] S.S.L. Cheng. L.A. Zadeh, On fuzzy mapping and conterol, IEEE Transactions on Systems, Man and Cybernetics 2 (1972) 30-34.

[14] Y.J. Cho, N.J. Huang, S.M. Kang, Nonlinear equations for fuzzy mapping in probabilistic normed spaces, Fuzzy Sets Syst. 110 (2000) 115-122.

[15] D. Dubois, H. Prade, Operations on fuzzy numbers, Journal of Systems Science 9 (1978) 613-626.

[16] D. Dubois, H. Prade, Fuzzy Sets and Systms: Theory and Application, Academic Press, New York, 1980.

[17] J. Fang, On nonlinear equations for fuzzy mapping in probabilistic normed spaces, Fuzzy Sets Syst. 131 (2002) 357-364.

[18] R. Goetschel, W. Voxman, Elementary calculus, Fuzzy Sets Syst. 18 (1986) 31-43.

[19] J. Ma, G. Feng, An approach to H_∞ control of fuzzy dynamic systems, Fuzzy Sets Syst. 137 (2003) 367-386.

[20] S. Nahmias, Fuzzy variables, Fuzzy Sets and Systems 12 (1978) 97-111.

[21] J.J. Nieto and R. Rodriguez-Lopez, Existence of extremal solutions for quadratic fuzzy equations. Fixed Point Theory and Applications 3 (2005) 321-342.

[22] M. Mosleh, Solution of Dual Fuzzy Polynomial Equations by Modified Adomian Decomposition Method, Fuzzy Inf. Eng. 1 (2013) 45-56,

[23] M. Otadi, M. Mosleh, Solution of fuzzy polynomial equations by modified Adomian decomposition method, Soft Computing 5 (2013) 45-56.

[24] M. Tavassoli Kajani, B. Asady, A. Hadi Vencheh, An iterative method for solving dual fuzzy nonlinear equations. Appl. Math. Comput. 167 (2005) 316-323.

[25] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Inform. Sci. 8 (1975) 199-249.

[26] L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338-353.

[27] H.J. Zimmermann, Fuzzy Sets Theory and its Application, Kluwer Academic Press, Dordrecht, (1991).

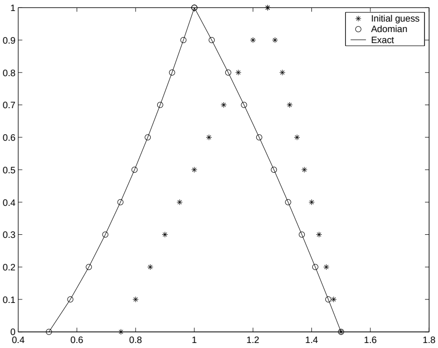

Fig. 1. Approximate and analytical solution of example 1 for x.

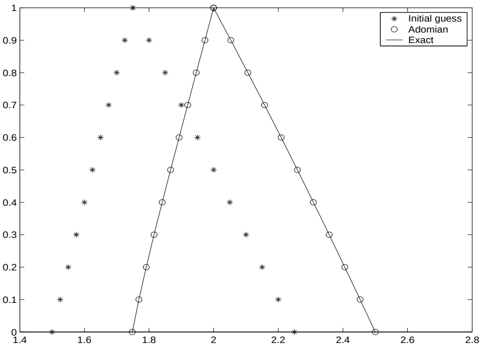

Fig. 2. Approximate and analytical solution of example 1 for y.

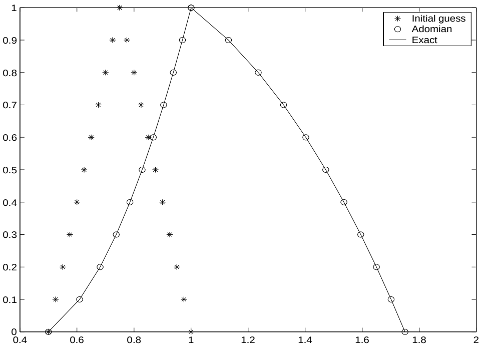

Fig. 3. Approximate and analytical solution of example 2 for x.

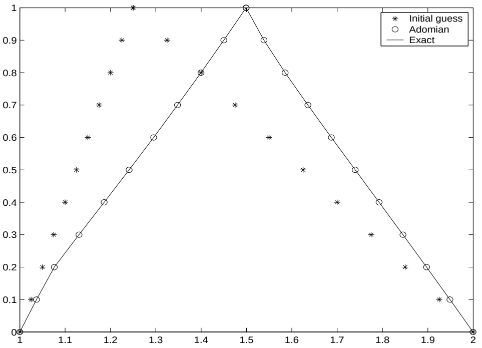

Fig. 4. Approximate and analytical solution of example 2 for y.

M	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5	Iter 6
0	0.2639	0.1336	0.0395	0.0357	0.0103	0.0100
1	0.1831	0.0422	0.0131	0.0092	2.6131×10^{-3}	2.4532×10^{-4}

Table 1. The error of Adomian decomposition method.

M	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5	Iter 6
0	1.4531	0.5312	0.1432	0.0339	0.0198	0.0114
1	0.4436	0.1253	0.0635	0.0092	1.5131×10^{-3}	3.464×10^{-4}

Table 2. The error of Adomian decomposition method.